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On the effective Lagrangian of QED with anomalous moments of the electron

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Abstract. The one-loop correction to the Maxwell Lagrangian with constant external field is obtained by taking into account the anomalous magnetic and electric moments of the electron. The imaginary part of the effective Lagrangian is found. It defines the probability of creating electron-positron pairs. The asymptotics of the effective Lagrangian for weak, $(H, E \ll m^2/e)$, and very strong, $(H, E \gg m^2/e)$, fields taking into account the dependence of anomalous moments of the electron on E, H are studied.

1. Introduction

It is known that taking the radiative corrections into account changes the value of the electron magnetic moment $\mu_0 = e/2m$† in Dirac's theory. To first order in the fine structure constant, α, the radiative corrections yield the anomalous magnetic moment (AMM), $\mu_1 = \mu_0\alpha/2\pi$ (Schwinger 1948). To second order in α, the AMM of the electron begins to depend on the external field (Newton 1954). This dependence becomes dominant for strong fields, $(E, H \sim m^2/e)$, when the AMM differs essentially from the Schwinger value (Ritus 1978).

The AMM of the electron, μ_1, can phenomenologically be taken into account in QED by adding the additional interaction of the Pauli form $\mu_1\bar{\psi}\sigma_{\mu\nu}F_{\mu\nu}\psi$ where $F_{\mu\nu}$ is the electromagnetic field tensor.

The AMM of the electron results in the corrections to the effective Schwinger Lagrangian (Schwinger 1951). These corrections for the uniform magnetic field, H, were found by Dittrich (1978).

The discovery of K-meson decays (Christenson *et al* 1964) makes us doubt the *CP* invariance hypothesis of physics equations (Landau 1957) because this decay is forbidden if *CP* parity is conserved. Breaking *CP* invariance leads to the conclusion that elementary particles can have electric moments. From this point of view the search for electric moments of particles and, if they exist, the study of their consequences is of fundamental importance. Note that taking into consideration the radiative corrections in QED with an intense constant electromagnetic field results in an anomalous electric moment (AEM) of the electron (Ritus 1978) if $(\boldsymbol{E}, \boldsymbol{H}) \neq 0$. The AEM of the electron, μ_2, can also phenomenologically be taken into account in QED by adding an additional interaction of the form $\mu_2\bar{\psi}\sigma_{\mu\nu}F^*_{\mu\nu}\psi$ where $2F^*_{\mu\nu} = \varepsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}$.

† The system of units in which $\hbar = c = 1$ is used. Dirac's matrices γ_μ satisfy the relations $\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}$, $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$, $2\sigma_{\mu\nu} = \mathrm{i}[\gamma_\mu, \gamma_\nu]$, $\gamma_5 = \mathrm{i}\gamma_0\gamma_1\gamma_2\gamma_3$, $\gamma_5^+ = \gamma_5$, $a_\mu b_\mu = a_0b_0 - a_ib_i$, $\{A, B\} = AB + BA$, $[A, B] = AB - BA$.

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The aim of this paper is to find and study the one-loop correction to the Maxwell Lagrangian for a constant electromagnetic field taking into account the AMM and AEM of the electron. Our treatment is based on Schwinger's coordinate-space and proper-time formalism (Schwinger 1951).

2. The effective Lagrangian

Our starting point is the Green function equation

$$(\hat{P}(x)+\mu\hat{\Phi}-m)G(x,x')=\delta(x-x')$$
$$\hat{P}(x)=\gamma_\mu P_\mu(x), \qquad P_\mu(x)=p_\mu-eA_\mu(x), \qquad \hat{\Phi}=\sigma_{\mu\nu}\Phi_{\mu\nu} \tag{2.1}$$
$$2\Phi_{\mu\nu}=F_{\mu\nu}\cos\delta+F^*_{\mu\nu}\sin\delta, \qquad \tan\delta=\mu_2/\mu_1, \qquad \mu=(\mu_1^2+\mu_2^2)^{1/2}$$

in which the AMM, μ_1 and AEM, μ_2, of the electron are phenomenologically taken into account. Later on the case of an arbitrary constant electromagnetic field is considered so that the field invariants

$$\mathscr{F}=\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu}=\tfrac{1}{2}(\boldsymbol{H}^2-\boldsymbol{E}^2), \qquad \mathscr{G}=\tfrac{1}{4}F^*_{\mu\nu}F_{\mu\nu}=(\boldsymbol{E},\boldsymbol{H})$$

might be arbitrary constants.

Equation (2.1) will be considered as the matrix equation

$$(\hat{P}+\mu\hat{\Phi}-m)G=1 \tag{2.2}$$

written in a non-physical Hilbert space of the abstract vectors, $|x\rangle$. Here

$$\langle x|x'\rangle=\delta(x-x'), \qquad \int \mathrm{d}x|x\rangle\langle x|=1, \qquad G(x,x')=\langle x|G|x'\rangle$$
$$\langle x|\hat{P}+\mu\hat{\Phi}-m|x'\rangle=(\hat{P}(x)+\mu\hat{\Phi}-m)\delta(x-x').$$

With the aid of the Green function (2.1) the one-loop correction, $\mathscr{L}(x)$, to the Maxwell Lagrangian is defined by the relations (Schwinger 1951)

$$S_1=\int \mathrm{d}x\,\mathscr{L}(x), \qquad \delta S_1=\mathrm{i}e\int \mathrm{d}x\,\mathrm{tr}\,\delta\hat{A}(x)G(x,x) \tag{2.3}$$

where tr indicates the diagonal summation in spinor space. Equation (2.3) in the matrix form is written in the following way

$$\delta S_1=\mathrm{i}e\,\mathrm{Tr}\,\delta\hat{A}G. \tag{2.4}$$

Here the trace Tr is taken in the functional sense. From (2.2), (2.3) and (2.4) in the standard way we obtain

$$S_1=\frac{\mathrm{i}}{2}\int_0^\infty\frac{\mathrm{d}s}{s}\exp(-\mathrm{i}m^2s)\,\mathrm{Tr}\exp(\mathrm{i}Hs) \tag{2.5}$$

up to an arbitrary additive constant and, therefore,

$$\mathscr{L}(x)=\frac{\mathrm{i}}{2}\int_0^\infty\frac{\mathrm{d}s}{s}\exp(-\mathrm{i}m^2s)\,\mathrm{tr}\langle x|\exp(\mathrm{i}Hs)|x\rangle. \tag{2.6}$$

Here the following notations are introduced

$$H=P^2-\tfrac{1}{2}e\hat{F}+\mu\{\hat{P},\hat{\Phi}\}+\mu^2\hat{\Phi}^2, \qquad \hat{F}=\sigma_{\mu\nu}F_{\mu\nu}. \tag{2.7}$$

Omitting the details of the calculations of the matrix element $\langle x|\exp(\mathrm{i}Hs)|x'\rangle$ we shall give the final expression for its diagonal part

$$\langle x|\exp(\mathrm{i}Hs)|x\rangle = -\frac{\mathrm{i}}{(4\pi)^2 s^2}\exp\left\{-\mathrm{i}s\left(\frac{e}{2}\hat{F} - \mu^2\hat{\Phi}^2 + \frac{\mu^2}{4}\{\gamma, \hat{\Phi}\}^2 - \tfrac{1}{2}\,\mathrm{Sp}\ln[(eFs)^{-1}\sinh(eFs)]\right)\right\} \tag{2.8}$$

where F denotes the matrix $F_{\mu\nu}$, Sp indicates the diagonal summation over vector indices of the matrices depending on F. Note that

$$\hat{\Phi}^2 = 2\Phi^2 + 2\mathrm{i}\Phi^*\Phi\gamma_5, \qquad \{\gamma, \hat{\Phi}\}^2 = 16\Phi^2$$

$$\Phi^2 = \mathscr{F}\cos 2\delta + \mathscr{G}\sin 2\delta, \qquad \Phi^*\Phi = \mathscr{G}\cos 2\delta - \mathscr{F}\sin 2\delta.$$

Then when calculating matrix traces in (2.6) and (2.8) suffice it to notice that the matrix F has the eigenvalues

$$\pm\mathrm{i}\sqrt{2}\,[(\mathscr{F}+\mathrm{i}\mathscr{G})^{1/2} + (\mathscr{F}-\mathrm{i}\mathscr{G})^{1/2}], \qquad \pm\mathrm{i}\sqrt{2}\,[(\mathscr{F}+\mathrm{i}\mathscr{G})^{1/2} - (\mathscr{F}-\mathrm{i}\mathscr{G})^{1/2}]$$

and the eigenvalues of the matrix $\frac{1}{2}e\hat{F} - 2\mathrm{i}\mu^2\Phi^*\Phi\gamma_5$ equal

$$\pm e\sqrt{2}\,(\mathscr{F}+\mathrm{i}\mathscr{G})^{1/2} - 2\mathrm{i}\mu^2\Phi^*\Phi, \qquad \pm e\sqrt{2}\,(\mathscr{F}-\mathrm{i}\mathscr{G})^{1/2} + 2\mathrm{i}\mu^2\Phi^*\Phi.$$

With the substitution $s \to -\mathrm{i}s$ in (2.6) we find the following expression for $\mathscr{L}(x)$

$$\mathscr{L}(x) = -\frac{1}{8\pi^2}\int_0^\infty \frac{\mathrm{d}s}{s^3}\exp\{-(m^2 + 2\mu^2\Phi^2)s\} \times\left((es)^2\mathscr{G}\cos(2\mu^2\Phi^*\Phi s)\frac{\mathrm{Re}\cosh(eXs)}{\mathrm{Im}\cosh(eXs)} - 1 - (es)^2\mathscr{G}\sin(2\mu^2\Phi^*\Phi s)\right) \tag{2.9}$$

where $X = \sqrt{2}\,(\mathscr{F}+\mathrm{i}\mathscr{G})^{1/2}$ and the additive constant in (2.9) is chosen so that when $\mathscr{F} = \mathscr{G} = 0 \Rightarrow \mathscr{L}(x) = 0$.

The Lagrangian $\mathscr{L}(x)$ (2.9) contains logarithmic divergences and on subtracting them we obtain the final expression for $\mathscr{L}(x)$

$$\mathscr{L}(x) = -\frac{1}{8\pi^2}\int_0^\infty \frac{\mathrm{d}s}{s^3}\exp\{-(m^2 + 2\mu^2\Phi^2)s\}\left((es)^2\mathscr{G}\cos(2\mu^2\Phi^*\Phi s)\frac{\mathrm{Re}\cosh(eXs)}{\mathrm{Im}\cosh(eXs)} - 1 - \tfrac{2}{3}(es)^2\mathscr{F} + (\mu^2 s)^2(\Phi^*\Phi)^2 - (es)^2\mathscr{G}\sin(2\mu^2\Phi^*\Phi s)\right). \tag{2.10}$$

If $\mathscr{F}$ and $\mathscr{G}$ do not vanish simultaneously, we may pass to the Lorenz frame in which electric and magnetic fields are parallel to each other

$$\mathscr{L}(x) = -\frac{1}{8\pi^2}\int_0^\infty \frac{\mathrm{d}s}{s^3}\exp\{-(m^2 + 2\mu^2\Phi^2)s\} \times[(es)^2EH\cos(2\mu^2\Phi^*\Phi s)\cot(eEs)\coth(eHs) - 1 - \tfrac{1}{3}(es)^2(H^2 - E^2) + (\mu^2 s)^2(\Phi^*\Phi)^2 - (es)^2EH\sin(2\mu^2\Phi^*\Phi s)], \tag{2.11}$$

$$2\mu^2\Phi^2 = (\mu_1 H + \mu_2 E)^2 - (\mu_1 E - \mu_2 H)^2, \qquad \mu^2\Phi^*\Phi = (\mu_1 H + \mu_2 E)(\mu_1 E - \mu_2 H).$$

From (2.10) when $\mu=0$ the well known result follows (Schwinger 1951) for the effective Lagrangian

$$\mathscr{L}(x)=-\frac{1}{8\pi^2}\int_0^\infty \frac{ds}{s^3}\exp(-m^2 s)\left((es)^2\mathscr{G}\frac{\operatorname{Re}\cosh(eXs)}{\operatorname{Im}\cosh(eXs)}-1-\tfrac{2}{3}(es)^2\mathscr{F}\right).$$

From (2.11) for a constant pure magnetic field, $(E\to 0)$, we have the form

$$\mathscr{L}(x)=-\frac{1}{8\pi^2}\int_0^\infty \frac{ds}{s^3}\exp\{-[m^2+(\mu_1^2-\mu_2^2)H^2]s\}$$
$$\times[(eHs)\cos(2\mu_1\mu_2H^2s)\coth(eHs)-1-\tfrac{1}{3}(eHs)^2+(\mu_1\mu_2H^2s)^2] \quad (2.12)$$

which when $\mu_2=0$ coincides with that of Dittrich (1978)

$$\mathscr{L}[H]=-\frac{1}{8\pi^2}\int_0^\infty \frac{ds}{s^3}\exp\{-(m^2+\mu_1^2H^2)s\}$$
$$\times[(eHs)\coth(eHs)-1-\tfrac{1}{3}(eHs)^2]. \quad (2.13)$$

Integral (2.11) when $E\neq 0$ has the poles at the points $S_n=\pi n/|e|E$. On integrating these poles should be passed round from above and we have the following contribution to the imaginary part of $\mathscr{L}$

$$2\operatorname{Im}\mathscr{L}=\frac{e^2EH}{4\pi^2}\sum_{n=1}^{\infty}\frac{1}{n}\coth\left(\pi n\frac{H}{E}\right)\cos\left(2\pi n\frac{\mu^2\Phi^*\Phi}{|e|E}\right)$$
$$\times\exp\left[-\frac{\pi n m^2}{|e|E}\left(1+\frac{2\mu^2\Phi^2}{m^2}\right)\right] \quad (2.14)$$

which defines the probability of electron–positron pair creation per unit time and per unit volume. In particular, when $H\to 0$ and $\mu_2=0$ from (2.14) we have

$$2\operatorname{Im}\mathscr{L}=\frac{e^2E^2}{4\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left[-\frac{\pi n m^2}{|e|E}\left(1-\frac{\mu_1^2E^2}{m^2}\right)\right] \quad (2.15)$$

and hence

$$\operatorname{Im}\mathscr{L}|_{\mu=0,H=0}<\operatorname{Im}\mathscr{L}|_{\mu_2=0,H=0}.$$

We see that the AMM of the electron leads to the increase of pair creation probability for a constant pure electric field. From (2.15) when $\mu_1=0$ the famous result obtained by Schwinger follows (Schwinger 1951).

3. Asymptotics of the effective Lagrangian

In this section we want to study limiting cases of the effective Lagrangian (2.10) or (2.11) for weak and very strong fields.

For weak fields, $(E\ll E_0,\ H\ll H_0,\ E_0,\ H_0=m^2/|e|)$ expression (2.10) admits asymptotic expansion in the parameters, E/E_0, H/H_0. In the first approximation

$$\mathscr{L}=\frac{e^4}{360\pi^2m^4}(4\mathscr{F}^2+7\mathscr{G}^2)+\frac{e^2}{8\pi^2m^2}\mathscr{G}[(\mu_1^2-\mu_2^2)\mathscr{G}-2\mu_1\mu_2\mathscr{F}] \quad (3.1)$$

the field invariants, $\mathscr{F}$ and $\mathscr{G}$, are present quadratically. In (3.1) the first term is the well known Heisenberg-Euler Lagrangian (Heisenberg and Euler 1936). As in modern experiments the electric moment, μ_2, is not discovered, then, in any case, for weak fields $\mu_2 \ll \mu_1$. Taking into account that for weak fields $\mu_1 = \mu_0\alpha/2\pi$, $\mu_0 = e/2m$ from (3.1) we obtain the modified Heisenberg-Euler Lagrangian

$$\mathscr{L} = \frac{e^4}{360\pi^2 m^4}\left[4\mathscr{F}^2 + \left(7 + 20\frac{\mu_1^2}{\mu_0^2}\right)\mathscr{G}^2\right] \tag{3.2}$$

caused by the AMM of the electron.

Consider now asymptotics of the effective Lagrangian (2.11) for very strong fields.

Let us confine ourselves first to the particular case for the effective Lagrangian (2.13) which will be written in the form

$$\begin{aligned}\mathscr{L}[H] &= -\frac{e^2H^2}{8\pi^2}\int_0^\infty \frac{\mathrm{d}x}{x}\exp\left\{-\left(\frac{m^2}{|e|H} + \frac{\mu_1^2 H}{|e|}\right)x\right\}\varphi(x),\\ \varphi(x) &= \frac{1}{x^2}[x\coth x - 1 - \tfrac{1}{3}x^2]\end{aligned} \tag{3.3}$$

by changing the variables $|e|Hs = x$. When $\mu_1 = 0$ and $H \gg H_0$ from (3.3) to a logarithmic accuracy we obtain well known asymptotics for the effective Lagrangian (Akhieser and Bereztetsky 1959)

$$\mathscr{L}_1[H] = \frac{e^2H^2}{24\pi^2}\ln\left(\frac{|e|H}{m^2}\right). \tag{3.4}$$

When studying the asymptotics of (3.3) one should take into account the AMM dependence of the electron on H. This dependence for very strong fields, $H \gg H_0$, has the form in the first approximation in $\chi \gg 1$ (Ritus 1978)

$$\mu_1 = \mu_0\frac{\alpha\Gamma(\frac{1}{3})}{81\chi^{2/3}}, \qquad \chi = \frac{H}{H_0}\left(\frac{p_\perp}{m}\right) \tag{3.5}$$

where $\Gamma(\frac{1}{3})$ is the value of the gamma-function at the point $\frac{1}{3}$, $p_\perp$ is the electron momentum to the direction perpendicular to the vector $\boldsymbol{H}$. From (3.3) and (3.5) when $H \gg H_0$ to a logarithmic accuracy we have

$$\mathscr{L}_2[H] = \frac{e^2H^2}{24\pi^2}\ln\left(\frac{\mu_1^2 H}{|e|}\right) = \frac{e^2H^2}{24\pi^2}\left[\ln\left(\frac{|e|H}{m^2}\right) + \ln\left(\frac{\mu_1^2 m^2}{e^2}\right)\right] \tag{3.6}$$

where for μ_1 one should take expression (3.5). The second term in (3.6) can be considered as the contribution to the asymptotics (3.4) due to the dependence of μ_1 on a constant magnetic field. This addition

$$\Delta\mathscr{L} = \frac{e^2H^2}{24\pi^2}\ln\left(\frac{\mu_1^2 m^2}{e^2}\right)$$

for $H \gg H_0$ has a negative sign so that $\mathscr{L}_1[H] > \mathscr{L}_2[H]$.

Note that taking into consideration the AMM dependence on an external field for very strong fields on studying the effective Lagrangian asymptotics is fundamentally important. Indeed, if one neglects the dependence of μ_1 on H, the effective Lagrangian asymptotics when $H \gg H_0$ has the form

$$\tilde{\mathscr{L}}_2[H] = \frac{e^4}{360\pi^2\mu_1^4} + O\left(\frac{|e|}{\mu_1^2 H}\right). \tag{3.7}$$

When obtaining (3.7) it was taken into account that $\varphi(0)=0$, $\varphi'(0)=-\frac{1}{45}$. Expression (3.7) differs in essence from (3.6). In view of the asymptotics (3.6) and (3.7) obtained here one should say that the study of the effective Lagrangian asymptotics without taking into account the dependence of μ_1 on H was done by Dittrich (1978) incorrectly and the asymptotics obtained there do not have the form (3.7). This is connected with the use of the expansions in the parameter, $\mu_1^2 H/|e|$, which for very strong magnetic field, H, is not small if μ_1 does not depend on H.

For a constant electric field, E ($H\to 0$), the effective Lagrangian (2.11) has the following form

$$\mathscr{L}[E]=-\frac{e^2E^2}{8\pi^2}\int_0^\infty \frac{dx}{x^3}\exp\left[-\frac{m^2}{|e|E}\left(1-\frac{\mu_1^2E^2}{m^2}\right)x\right](x\cot x-1+\tfrac{1}{3}x^2) \tag{3.8}$$

by changing the variable $|e|Es=x$. When $E\gg E_0$ to a logarithmic accuracy for the real part of $\mathscr{L}[E]$ one obtains (Akhieser and Bereztetsky 1959)

$$\operatorname{Re}\mathscr{L}_1[E]=-\frac{e^2E^2}{24\pi^2}\ln\left(\frac{|e|E}{m^2}\right) \tag{3.9}$$

if $\mu_1=0$. Expression (3.9) results in the negative addition to the energy density, $\varepsilon_0=E^2/8\pi$, so that for a very strong electric field, E, the energy density, ε, equals

$$\varepsilon=\frac{E^2}{8\pi}\left[1-\frac{\alpha}{3\pi}\exp\left(\frac{|e|E}{m^2}\right)\right].$$

This density vanishes when

$$E=E_{\max}=E_0\exp(3\pi/\alpha)\sim 10^{560}E_0.$$

On these grounds it was concluded that (Greenman and Rohrlich 1973) $E_{\max}$ is the maximum electrostatic field strength in the universe. This conclusion should be considered groundless within the framework of QED because of the well known 'zero-charge' situation. The asymptotics over very strong external field can be established only in the interval (Voronov and Kryuchkov 1979)

$$1\ll E/E_0\ll\exp(3\pi/\alpha) \tag{3.10}$$

and, hence, any conclusions about an asymptotic behaviour of physics quantities beyond the interval (3.10) are not correct. Note that expression (3.8) makes sense if

$$\mu_1^2E^2<m^2. \tag{3.11}$$

Consider condition (3.11) for a very strong electric field, $E\gg E_0$ and take into account the dependence of μ_1 on E (Ritus 1978)

$$\mu_1=\mu_0\frac{\alpha\Gamma(\frac{1}{3})}{81\chi^{2/3}},\qquad \chi=\frac{E}{E_0}\left(1+\frac{p_\perp^2}{m^2}\right)^{1/2},\qquad p_\perp\neq 0. \tag{3.12}$$

This accounts for the fact that condition (3.11) may break when

$$E=E_{\max}=E_0\left(\frac{162}{\alpha\Gamma(\frac{1}{3})}\right)^4\left(1+\frac{p_\perp^2}{m^2}\right)\sim 10^{12}E_0\left(1+\frac{p_\perp^2}{m^2}\right).$$

For a very strong electric field, $E\gg E_0$ the AMM of the electron equals twice the Schwinger value (Ritus 1978)

$$\mu_1 = \mu_0 \alpha / \pi, \qquad p_\perp = 0 \tag{3.13}$$

and, hence, condition (3.11) breaks when

$$E = E_{\max} = E_0 2\pi/\alpha \sim 10^3 E_0.$$

For a constant electric field, E, the asymptotic behaviour of the effective Lagrangian taking into account the AMM of the electron can be established in the interval

$$1 \ll E/E_0 < 2\pi/\alpha \ll \exp(3\pi/\alpha). \tag{3.14}$$

The electric field strength, $E_{\max} = 2\pi E_0/\alpha$, should be considered as a maximum value when phenomenological consideration of the electron AMM in QED is contradictory.

When fulfilling condition (3.11) for a very strong electric field we obtain for the real part of $\mathscr{L}[E]$ (3.8)

$$\mathrm{Re}\,\mathscr{L}_2[E] = -\frac{e^2E^2}{24\pi^2}\left[\ln\left(\frac{|e|E}{m^2}\right) + \ln\left(1 - \frac{\mu_1^2E^2}{m^2}\right)^{-1}\right].$$

Note that the condition $\mu_2 = 0$ used above for studying the effective Lagrangian asymptotics for very strong magnetic ($E \to 0$) and electric ($H \to 0$) fields does not limit our consideration if the AEM of the electron is caused solely by radiative corrections due to an external field. Indeed, in this case $\mu_2 = 0$ because $\mathscr{G} = 0$ (Ritus 1978). For $\chi \gg 1$

$$\chi = \left[\frac{H^2}{H_0^2}\frac{p_\perp^2}{m^2} + \frac{E^2}{E_0^2}\left(1 + \frac{p_\perp^2}{m^2}\right)\right]^{1/2} \tag{3.15}$$

the AEM of the electron behaves in the following way (Ritus 1978)

$$\mu_2 = \mu_0 \frac{5\alpha}{3\pi}\left(\frac{EH}{E_0H_0}\right)\chi^{-2}\ln\left(\frac{\chi}{\sqrt{3}\,\gamma}\right), \qquad \ln\gamma = C \tag{3.16}$$

where C is Euler's constant, $C = 0.577$.

From (3.14) and (3.15) it follows that for very strong fields the AEM of the electron may take a value visibly different from zero only when $H \sim E$. In view of this let us consider the case when $E = H$. Then from (2.11) we obtain

$$\mathscr{L}[E = H] = -\frac{e^2E^2}{8\pi^2}\int_0^\infty \frac{\mathrm{d}x}{x^3}\exp(-Ax)$$
$$\times[x^2\cos(Bx)\coth x \cot x - 1 + B^2x^2 - x^2\sin(Bx)]. \tag{3.17}$$

Here the notation is introduced:

$$A = \frac{E_0}{E}\left[1 + \frac{\mu_1\mu_2}{\mu_0^2}\left(\frac{E}{E_0}\right)^2\right], \qquad B = 2\frac{E}{E_0}\left[\left(\frac{\mu_1}{\mu_0}\right)^2 - \left(\frac{\mu_2}{\mu_0}\right)^2\right]. \tag{3.18}$$

From (3.12) (or (3.13)) and (3.16) it follows that in the whole region of the values, E/E_0, from the interval (3.14) the parameters A, $|B| \ll 1$. Consequently, the asymptotic behaviour of the effective Lagrangian for very strong fields and satisfying condition (3.14) is defined from (3.17) at small values of the parameters, A, B. In particular, from (3.17) when $\mu_1 = \mu_2 = 0$ we obtain the asymptotics of the Schwinger Lagrangian real part for the fields, $E = H$, $E \gg E_0$

$$\mathrm{Re}\,\mathscr{L}_1[E = H] = \frac{m^4}{16\pi^2}\ln\left(\frac{|e|E}{m^2}\right).$$

In the general case the asymptotics of the real part (3.17) to a logarithmic accuracy has the form

$$\mathrm{Re}\,\mathscr{L}_2[E=H]=\frac{e^2E^2}{16\pi^2}(A^2-B^2)\ln\left(\frac{1}{A}\right)$$

where for the quantities, A, B, one should take the expression (3.18) and for μ_1 and μ_2 expressions (3.12) (or (3.13)) and (3.16) respectively.

4. Conclusion

In this paper the exact expression for the one-loop effective Lagrangian in QED taking into consideration the anomalous moments of the electron is obtained. It is established there there is a modification of the well known Heisenberg-Euler Lagrangian for weak fields. The imaginary part of the effective Lagrangian which defines the probability of pair creation is found. The asymptotics of the effective Lagrangian for very strong fields are found. The asymptotic calculations point to the importance of taking into account the dependence of anomalous moments of the electron on external fields. The region of field strength values in which the obtained asymptotic expressions for the effective Lagrangian are true is found.

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